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Structural Displacement Estimation by FIR Filter Based Fusion of Strain and Acceleration Measurements

Zhanxiong Ma and Hoon Sohn
Department of Civil and Environmental Engineering, Korea Advanced Institute of Science and Technology
Daejeon, Korea

ABSTRACT

In this study, a FIR filter based displacement estimation technique is proposed to estimate displacement from distributed strain sensors and an accelerometer. First, the neutral axis location of a structure is estimated from strain and acceleration measurements using recursive least square (RLS) method. Then, the low frequency displacement is estimated from the distributed strain gauges and the high frequency displacement from the accelerometer. Finally, these low and high frequency displacements are combined for high fidelity displacement estimation. The feasibility of the proposed method is examined by numerical simulation of a simply-supported beam model under low-frequency and random noise excitations.

KEY WORDS: Displacement estimation; FIR filter; Neutral axis location; Strain gauge; Accelerometer; Data fusion.

INTRODUCTION

Displacement plays a vital role in structural health monitoring as it provides crucial information regarding structural integrity and its current condition. For example, the displacement response has been employed to identify structural modal parameters of built-up structures (Kim, Kim and Sohn, 2013) and these identified parameters can be used as structural damage indicators.

Although the direct measurement of the displacement response is quiet challenging, there are several techniques available. For instance, linear variable differential transformer (LVDT) was applied to measure displacement response of timber railroad bridges (Moreu, Jo, Li, Kim, Cho, Kimmle, Scola, Le, and Spencer, 2013). While the displacement response can be measured with high accuracy and high sampling rate, the installation of LVDT is quite time-consuming. Besides, a fixed support is required as the reference point, which may be impossible in some applications. The global positioning system (GPS) also can be applied for the displacement measurement. This technique was adopted to measure wind-induced displacements of the suspension bridge girder (Nakamura, 2000) and building structures (Breuera, Chmielewski, Górski, and Konopka, 2002; Tamura, Matsui, Pagnini, Ishibashi, and Yoshida, 2002). Although GPS-type technique does not require a fixed reference point, it usually suffers from the low sampling rate and low accuracy problems. Moreover, this technique cannot work under GPS denied environment like underwater conditions.

Much research has focused on developing displacement estimation techniques by fusing different types of sensors to overcome limitations of direct measurement techniques. For example, A two-stage Kalman filter was proposed to combine RTK-GPS and the accelerometer (Kim, Choi, Chung, Koo, Bae, and Sohn, 2014; Koo, Kim, Chung, Choi, Kwon, Kang, and Sohn, 2017). With the help of the acceleration information, the sampling rate of estimated displacement can be significantly increased to 100 Hz, which is basically enough for civil engineering application. In addition, the accuracy can be improved to millimeter level as well. However, one remaining issue is that this technique still cannot work in GPS denied environment. Another attempt was done by using a FIR filter to fusing strain gauges and accelerometer (Shin, Lee, Kim, and Kim, 2012). The high frequency displacement calculated from the acceleration measurement and the low frequency displacement converted from strain measurements were combined to obtain the displacement estimate with high accuracy. However, when converting strain measurements to the displacement, prior knowledge of the neutral axis location is required, which is determined by calibration experiments in some cases. Although a power spectral density (PSD) based method was suggested to estimate the neutral axis location from strain and acceleration measurements (Park, Sim, and Jung, 2013), it is still not convenient since strain and acceleration measurements have to be transformed from time domain into frequency domain. Furthermore, the estimated neutral axis location suffers from the low accuracy problem especially when the noise level is high in measurements.

In this study, a FIR filter based approach is proposed for the displacement estimation by fusing measurements obtained from distributed strain sensors and single accelerometer. Firstly, the location of neutral axis is estimated based on the strain and acceleration measurements by utilizing recursive least square (RLS) method. Then, the strain measurements are converted to the displacement with estimated neutral axis location. Finally, the high-fidelity displacement estimate is acquired by adopting the FIR filter to fuse acceleration measurement with converted displacement from strain measurements.

The paper is organized as follows: the basic principle of the proposed displacement estimation technique is introduced in Theory Background section. Then, Numerical Verification section examines the performance by using a simply-supported beam model. Finally, the concluding remarks are provided in Conclusion section.

THEORY BACKGROUND

The basic principle of proposed displacement estimation approach is briefly illustrated in this section. Suppose that we have an Euler–Bernoulli beam as shown in Fig 1, and strain responses are measured at multiple points. Firstly, the displacement at the target location can be converted from strain measurements using following equations (Shin, Lee, Kim, and Kim, 2012):

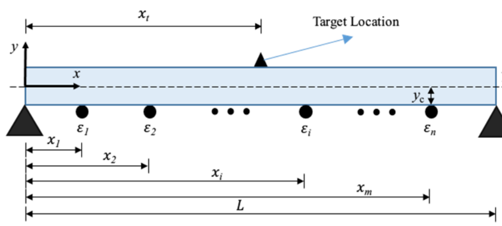


Fig.1 Euler–Bernoulli beam

$$u_c(x_t,t) = \frac{1}{y_c}\boldsymbol{\Psi}(x_t)\left\{\boldsymbol{\Phi}(x)^T\boldsymbol{\Phi}(x)\right\}^{-1}\boldsymbol{\Phi}(x)^T\boldsymbol{\varepsilon}(x,t) \tag{1}$$

$$\boldsymbol{\Psi}(x_t) = \begin{bmatrix}\sin(\frac{\pi x_t}{L}) & \cdots & \sin(\frac{\pi x_t}{L})\end{bmatrix} \tag{2}$$

$$\boldsymbol{\Phi}(x) = \frac{\pi^2}{L^2}\begin{bmatrix}\sin(\frac{\pi x_1}{L}) & \cdots & r^2\sin(\frac{r\pi x_1}{L}) \\ \vdots & \ddots & \vdots \\ \sin(\frac{\pi x_n}{L}) & \cdots & r^2\sin(\frac{r\pi x_n}{L})\end{bmatrix} \tag{3}$$

$$\boldsymbol{\varepsilon}(x,t) = \begin{bmatrix}\varepsilon(x_1,t) & \cdots & \varepsilon(x_n,t)\end{bmatrix}^T \tag{4}$$

where: u_c is the converted displacement at $x = x_t$; x_t is the displacement estimation location in x direction; x_i are locations of strain sensors in x direction ($i = 1,\cdots,n$); n is the number of strain sensors; r is the number of modes considered. L is the length of the beam. y_c is the distance from the strain sensor location to the neutral axis in y direction; $\varepsilon(x_i, t)$ is the strain measurement at $x = x_t$.

In practice, it is quite difficult to determine the location of the neutral axis. Thus, a RLS based method is proposed in this study for estimating neutral axis location. According to the physical relationship, the displacement can be easily calculated from double integration of the acceleration. Although the existence of the noise in the acceleration measurement and unknown initial conditions (initial velocity and initial displacement) will cause large low frequency drift, displacement calculated from the acceleration has a good accuracy in the high frequency part. It means that the high frequency displacement estimate can be easily obtained with high accuracy from acceleration measurement by adopting a high-pass filter. On the other hand, if the same high-pass filter is applied to the displacement converted from strain measurements, we can have another high frequency displacement estimate. Obviously, two displacement estimates should be identical if the value of y_c is set accurately. Based on this relationship, y_c will be identified by RLS method.

After the neutral axis location is identified, the converted displacement with the estimated y_c and the acceleration measurement will be inputted into the FIR filter which was firstly proposed by Lee, Hong, and Park (2010) and later improved by Hong, Lee, and Lee (2013). The final estimated displacement can be expressed as follow:

$$\boldsymbol{u}_e = (dt)^2(\boldsymbol{L}^T\boldsymbol{L} + \lambda^2\boldsymbol{I})^{-1}\boldsymbol{L}^T\boldsymbol{L}_a\boldsymbol{a} + \lambda^2(\boldsymbol{L}^T\boldsymbol{L} + \lambda^2\boldsymbol{I})^{-1}\boldsymbol{u}_c \tag{5}$$

where: $\boldsymbol{u}_e$, $\boldsymbol{u}_c$, and $\boldsymbol{a}$ is estimated displacement, converted displacement from strain measurements, and acceleration measurement, respectively; dt is the sampling time. More details can be found in Lee, Hong, and Park (2010) and Hong, Lee, and Lee (2013).

NUMERICAL VERIFICATION

A uniform simply-supported beam model as shown in Fig. 1, was built in SAP2000 to examine the proposed displacement estimation technique. The model is composed of ten Euler-Bernoulli beam elements, each of which has the length of 1m and a square cross-section of 120mm width. Three strain gauges were installed at points A, B and C, while one accelerometer was installed at point B. Sensor layout is shown in Fig 2

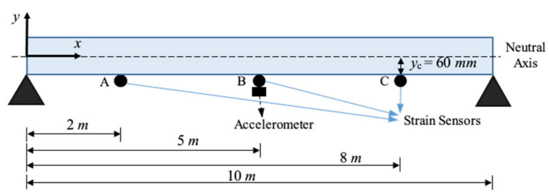
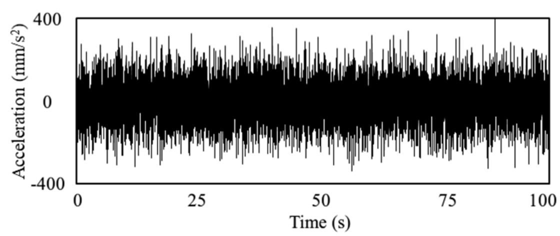
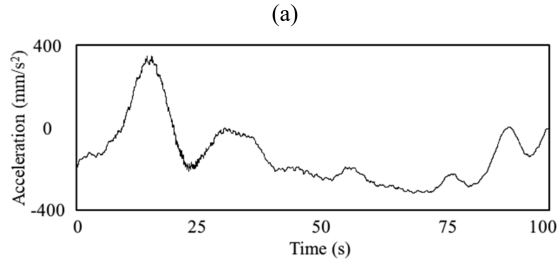


Fig.2 Simply-supported Euler beam model



(a)

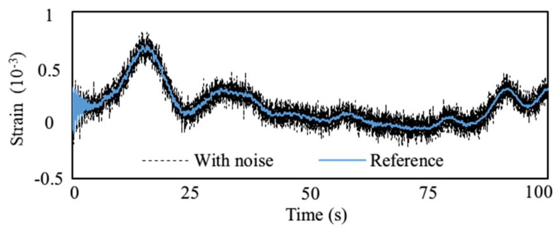


(b)

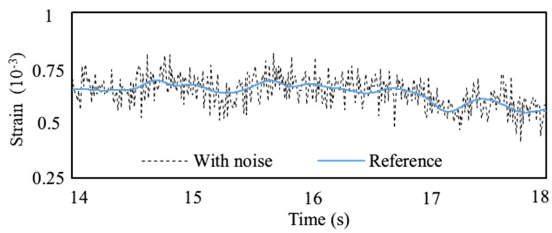
Fig.3 Excitation signals for: (a) Case 1, and (b) Case 2

The model was excited by the ground motion in terms of the acceleration signal. Two different excitation signals are considered here: one is the random noise with frequency range [0-50] Hz, the other is low frequency components (below 0.1Hz) dominant signal. The plots of excitation signals are shown in Fig. 3.

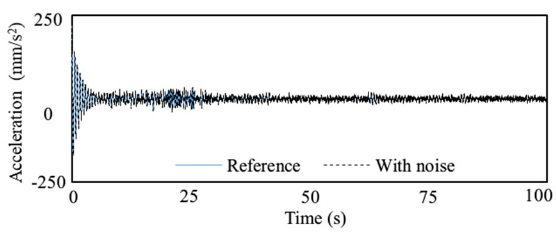
Time history analysis was conducted in SAP2000 with two different excitations to simulate acceleration and strain responses for displacement estimation as well as displacement response as reference data for accuracy evaluation. All simulated signals are sampled at 100 Hz. It should be noted that simulated strain and acceleration responses are polluted manually by adding different white noises. Considering that accelerometers usually have a higher SNR than strain gauges in real application, the signal noise ratios (SNR) is set as 10 for the strains responses and 20 for acceleration response, respectively. Fig. 4 shows the difference between original simulated and polluted responses at point B with excitation case 2.



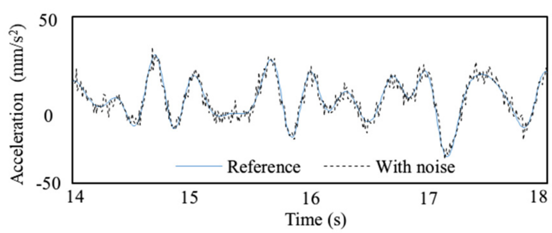
(a)



(b)



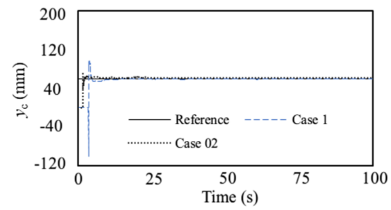
(c)



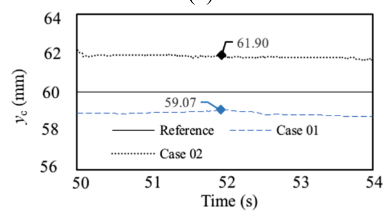
(d)

Fig.4 Measurements at point A with case 2: (a) Strain, (b) Strain (Zoomed in [14s-18s]), (c) Acceleration, and (d) Acceleration (Zoomed in [14s-18s])

Fig. 5 represents the result of the estimated y_c. Since it takes time for RLS method to converge, the huge discrepancy could be observed at the first several seconds. However, with the lapse of time, the estimated values of y_c became very close to the reference value which is 60*mm*. For example, at t = 52*s*, the estimated y_c was 59.07*mm* for case 1 and 61.90*mm* for case 2, which has only 1.55% and 3.17% error respectively.



(a)



(b)

Fig.5 Estimated neutral axis locations with two different excitations: (a) whole time period, and (b) zoomed in [50s- 54s]

Mean values of estimated neutral axis locations were calculated in Table 1, and errors are only 1.60% for case 1 and 3.20% for case 2. It should be noted that mean values were calculated based on the data from 10*s* to 100*s* considering that huge discrepancy observed in first several seconds.

Table 1 Accuracy comparison of estimated neutral axis locations with two different excitations

	Reference (mm)	Mean of estimated y_c (mm)	Error (%)
Case 1	60	59.03	1.60
Case 2		61.92	3.20

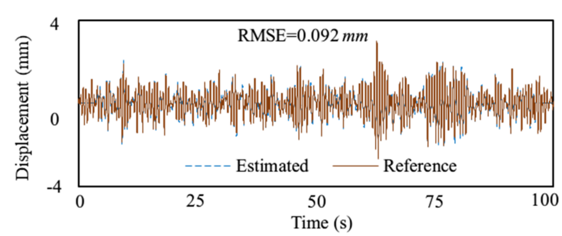
The estimated displacements also show a good agreement with the reference displacements in both cases, which can be observed from the plots shown in Fig. 6. The performance of the proposed method was evaluated using absolute peak value error (APVE) and the root mean square error (RMSE) which are defined by Eqs. 6~7,

$$APVE = \left| \frac{\max(|u_e|) - \max(|u_r|)}{\max(|u_r|)} \right| \tag{6}$$

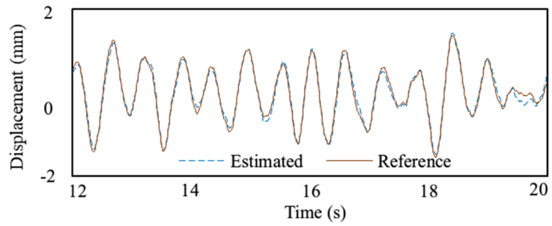
$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(u_e - u_r)^2}{N-1}} \tag{7}$$

where: u_e and u_r are the estimated and the reference displacement respectively; N is the number of time steps.

Table 2 shows the evaluation result. Note that APVE and RMSE are calculated based on the data from 10*s* to 90*s*, because the estimated displacements are not available for the first and the last several seconds in FIR filter. It is clear that the proposed method has good performance with APVE smaller than 3% and RMSE smaller than 0.1*mm*.



(a)



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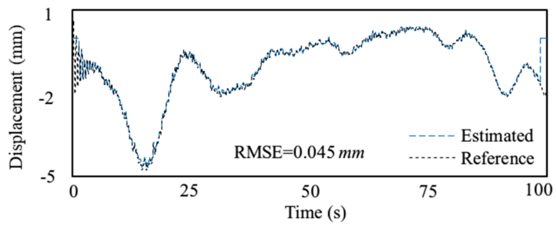

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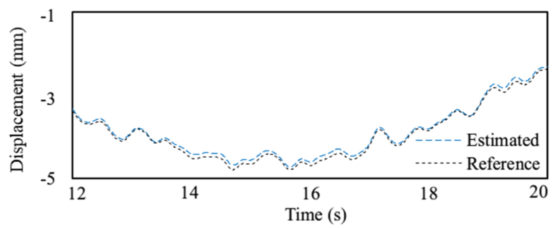

(d)

Fig.6 Displacements estimated by proposed method under different excitations: (a) Case 1, (b) Case 1 (Zoomed in [12s-20s]), (c) Case 2, and (d) Case 2 (Zoomed in [12s-20s])

Table 2 Accuracy comparison of displacements estimated by proposed method under different excitation cases

	Absolute Peak Value			RMSE (mm)
	Reference (mm)	Estimated (mm)	Error (%)	
Case 1	3.002	2.919	2.77	0.092
Case 2	4.786	4.717	1.45	0.045

CONCLUSIONS

This study presents a FIR-filter based displacement estimation technique which estimates high accuracy displacement by combining strain and acceleration measurements. The simulation results show that the RLS method can effectively estimate the location of the neutral axis with high accuracy (less than 4% error). The results also show that the proposed method can estimate precise displacement with RMSE smaller than 0.1 *mm*. However, only a simply supported beam model is considered in this study and real structures are more complicated, so lab-scale experiments and field tests are required to further verification.

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